

Fig. 2 Comparison between exact pressure distribution and first approximation (integration regions hatched).

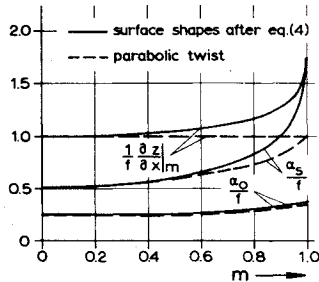


Fig. 3 Angles of attack for vanishing leading-edge singularity and for zero lift.

singularity does not change noticeably (see below), the first approximation for a cambered wing is better than for a flat one.

The case of zero (or finite) leading-edge pressure can be obtained from Eq. (2) by requiring the singular part to vanish. From this, the proper angle of attack is obtained for shapes after Eq. (4) as

$$\alpha_s/f = c/f[1 + (1 - \sqrt{1 - m^2})K'(m) - E'(m)] \quad (5)$$

Also, for parabolic twist this angle can be expressed analytically as

$$\alpha_s/f = 2/\pi K(m)K'(m) + 2/m^2 \{1 - 1/\pi E'(m)[E(m) + K(m)]\} \quad (6)$$

with  $K(m)$  and  $E(m)$  being the complete elliptic integrals of the first and second kind respectively. The angle of attack for zero lift follows from Eq. (2) by integrating the nonsingular part to find  $C_{L_s}$  and using the well-known result for the lift curve slope<sup>1</sup>

$$\alpha_0/f = \alpha_s/f - C_{L_s}/(fC_{L_\alpha}) = c/f\{1 - E'(m)/2 + (1 - \sqrt{1 - m^2})[K'(m) - E'(m)/m^2]\} \quad (7)$$

For parabolic twist this characteristic angle has to be evaluated numerically, except for the limiting cases:

$$\text{for } m=0 \quad \alpha_0/f = 1/4$$

$$\text{for } m=1 \quad \alpha_0/f = 1/3$$

In Fig. 3 the characteristic angles of attack are shown and the streamwise contour slopes at the leading edge are also plotted for comparison. The typical increase of  $\alpha_s$  (vanishing leading-edge singularity) for  $m \rightarrow 1$  can be seen. In the case of parabolic twist this is due solely to the diminishing streamline slope in front of the leading edge and remains remarkably lower than that in the case of Eq. (4). On the other hand, the zero-lift angles of attack differ only by a small amount. Furthermore, all differences between the exact solution and the first approximation discussed above are too small to be plotted in this figure.

More details of the analysis can be found in Ref. 6.

## References

- <sup>1</sup>Carafoli, E., *High-Speed Aerodynamics*, Pergamon Press, London/New York/Paris, 1956, pp. 619-624.
- <sup>2</sup>Holla, V. S. and Krishnaswamy, T. N., "Conically Cambered Triangular Wings," *Journal of the Aeronautical Society of India*, Vol. 22, May 1970, pp. 94-107.
- <sup>3</sup>Bera, R. K., "Slender Delta Wings with Conical Camber," *Journal of Aircraft*, Vol. 11, April 1974, pp. 245-247.
- <sup>4</sup>Evvard, J. C., "Effects of Yawing Thin Pointed Wings at Supersonic Speeds," NACA TN 1429, 1947.
- <sup>5</sup>Hancock, G. J., "Note on the Extension of Evvard's Method to Wings with Subsonic Leading Edges Moving at Supersonic Speeds," *Aeronautical Quarterly*, Vol. VIII, 1957, pp. 87-102.
- <sup>6</sup>Wagner, B., "Untersuchungen über konische Überschalltragflächen mit Unterschallvorderkanten," Institut für Flugtechnik, T. H. Darmstadt, Rept. No. 8/75, 1975.

## Errata

### Remarks on Thin Airfoil Theory

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IN the above titled Note, the general solution for the integral

$$J_n(\phi) = \int_0^\pi \frac{\sin n\theta}{\cos\theta - \cos\phi} d\theta \quad (1)$$

is found to be in error. The integral has the recurrence relation

$$J_{n+1} + J_{n-1} = 2\cos\phi J_n + (2/n)[1 - (-1)^n] \quad (2)$$

with starting values

$$J_0 = 0 \quad (3)$$

$$J_1 = 2 \log \tan(\phi/2) \quad (4)$$

The subsequent derivation in Ref. 1 for finding a general solution for Eq. (2) appears to be in error. The correct solution is determined to be (for  $n \geq 1$ )

$$J_n = 2 \log \tan \frac{\phi}{2} \frac{\sin n\phi}{\sin \phi} + \frac{2}{\sin \phi} \sum_{k=1}^{n-1} \frac{[1 - (-1)^k]}{k} \sin(n-k)\phi \quad (5)$$

and may be verified by substitution in Eq. (2). The final solution, Eq. (5), was written down by inspection after obtaining  $J_0$  to  $J_{10}$  in explicit form using Eqs. (2-4).

For computational efficiency it is preferable to use Eqs. (2-4), since it avoids a large number of trigonometric function evaluations that would otherwise be required if Eq. (5) is used.

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Index category: Aircraft Aerodynamics (including Component Aerodynamics).

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